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> restart
> Sistema := diff(x(t), t) = -x(t) + y(t) + z(t) + exp(t), diff(y(t), t) = x(t) - y(t) + z(t)
+ exp(3 t), diff(z(t), t) = x(t) + y(t) + z(t) + 4 : Sistema[1]; Sistema[2]; Sistema[3]

$$\frac{d}{dt} x(t) = -x(t) + y(t) + z(t) + e^t$$


$$\frac{d}{dt} y(t) = x(t) - y(t) + z(t) + e^{3t}$$


$$\frac{d}{dt} z(t) = x(t) + y(t) + z(t) + 4 \quad (1)$$

=
> CondIni := x(0) = 6, y(0) = -2, z(0) = 10
CondIni := x(0) = 6, y(0) = -2, z(0) = 10 \quad (2)
=
> AA := array([[-1, 1, 1], [1, -1, 1], [1, 1, 1]])

$$AA := \begin{bmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \quad (3)$$

=
> BB := array([exp(t), exp(3 t), 4])

$$BB := \begin{bmatrix} e^t & e^{3t} & 4 \end{bmatrix} \quad (4)$$

=
> Cond := array([6, -2, 10])

$$Cond := \begin{bmatrix} 6 & -2 & 10 \end{bmatrix} \quad (5)$$

=
> with(linalg) :
> MatExp := exponential(AA, t); MatExp[1, 1]; MatExp[3, 3]

$$MatExp := \begin{bmatrix} \frac{e^{-t}}{3} + \frac{e^{2t}}{6} + \frac{e^{-2t}}{2} & \frac{e^{-t}}{3} - \frac{e^{-2t}}{2} + \frac{e^{2t}}{6} & \frac{e^{2t}}{3} - \frac{e^{-t}}{3} \\ \frac{e^{-t}}{3} - \frac{e^{-2t}}{2} + \frac{e^{2t}}{6} & \frac{e^{-t}}{3} + \frac{e^{2t}}{6} + \frac{e^{-2t}}{2} & \frac{e^{2t}}{3} - \frac{e^{-t}}{3} \\ \frac{e^{2t}}{3} - \frac{e^{-t}}{3} & \frac{e^{2t}}{3} - \frac{e^{-t}}{3} & \frac{2e^{2t}}{3} + \frac{e^{-t}}{3} \end{bmatrix}$$


$$\frac{e^{-t}}{3} + \frac{e^{2t}}{6} + \frac{e^{-2t}}{2}$$


$$\frac{2e^{2t}}{3} + \frac{e^{-t}}{3} \quad (6)$$

=
> SolHom := evalm(MatExp &* Cond); x(t) = SolHom[1]; y(t) = SolHom[2]; z(t) = SolHom[3]
SolHom :=  $\begin{bmatrix} -2e^{-t} + 4e^{2t} + 4e^{-2t} & -2e^{-t} - 4e^{-2t} + 4e^{2t} & 8e^{2t} + 2e^{-t} \end{bmatrix}$ 

$$x(t) = -2e^{-t} + 4e^{2t} + 4e^{-2t}$$


$$y(t) = -2e^{-t} - 4e^{-2t} + 4e^{2t}$$


$$z(t) = 8e^{2t} + 2e^{-t} \quad (7)$$

=
> CompUno := x(0) = simplify(subs(t=0, SolHom[1]))

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$$CompUno := x(0) = 6 \quad (8)$$

$$\begin{aligned} &> CompDos := y(0) = \text{simplify}(\text{subs}(t=0, SolHom[2])) \\ &CompDos := y(0) = -2 \end{aligned} \quad (9)$$

$$\begin{aligned} &> CompTres := z(0) = \text{simplify}(\text{subs}(t=0, SolHom[3])) \\ &CompTres := z(0) = 10 \end{aligned} \quad (10)$$

$$\begin{aligned} &> CondIni \\ &x(0) = 6, y(0) = -2, z(0) = 10 \end{aligned} \quad (11)$$

$$\begin{aligned} &> MatExpTau := \text{map}(\text{rcurry}(\text{eval}, t=t - \text{tau}'), MatExp) \\ &MatExpTau := \end{aligned} \quad (12)$$

$$\begin{bmatrix} \frac{e^{-t+t}}{3} + \frac{e^{2t-2t}}{6} + \frac{e^{-2t+2t}}{2} & \frac{e^{-t+t}}{3} - \frac{e^{-2t+2t}}{2} + \frac{e^{2t-2t}}{6} & \frac{e^{2t-2t}}{3} - \frac{e^{-t+t}}{3} \\ \frac{e^{-t+t}}{3} - \frac{e^{-2t+2t}}{2} + \frac{e^{2t-2t}}{6} & \frac{e^{-t+t}}{3} + \frac{e^{2t-2t}}{6} + \frac{e^{-2t+2t}}{2} & \frac{e^{2t-2t}}{3} - \frac{e^{-t+t}}{3} \\ \frac{e^{2t-2t}}{3} - \frac{e^{-t+t}}{3} & \frac{e^{2t-2t}}{3} - \frac{e^{-t+t}}{3} & \frac{2e^{2t-2t}}{3} + \frac{e^{-t+t}}{3} \end{bmatrix}$$

$$\begin{aligned} &> BB[2] \\ &e^{3t} \end{aligned} \quad (13)$$

$$\begin{aligned} &> BBtau := \text{map}(\text{rcurry}(\text{eval}, t=t - \text{tau}'), BB) \\ &BBtau := \begin{bmatrix} e^t & e^{3t} & 4 \end{bmatrix} \end{aligned} \quad (14)$$

$$\begin{aligned} &> ProdTau := \text{evalm}(MatExpTau \&* BBtau) : ProdTau[1]; ProdTau[2]; ProdTau[3] \\ &\left( \frac{e^{-t+t}}{3} + \frac{e^{2t-2t}}{6} + \frac{e^{-2t+2t}}{2} \right) e^t + \left( \frac{e^{-t+t}}{3} - \frac{e^{-2t+2t}}{2} + \frac{e^{2t-2t}}{6} \right) e^{3t} + \frac{4e^{2t-2t}}{3} \\ &\quad - \frac{4e^{-t+t}}{3} \\ &\left( \frac{e^{-t+t}}{3} - \frac{e^{-2t+2t}}{2} + \frac{e^{2t-2t}}{6} \right) e^t + \left( \frac{e^{-t+t}}{3} + \frac{e^{2t-2t}}{6} + \frac{e^{-2t+2t}}{2} \right) e^{3t} + \frac{4e^{2t-2t}}{3} \\ &\quad - \frac{4e^{-t+t}}{3} \\ &\left( \frac{e^{2t-2t}}{3} - \frac{e^{-t+t}}{3} \right) e^t + \left( \frac{e^{2t-2t}}{3} - \frac{e^{-t+t}}{3} \right) e^{3t} + \frac{8e^{2t-2t}}{3} + \frac{4e^{-t+t}}{3} \end{aligned} \quad (15)$$

$$\begin{aligned} &> SolNoHom := \text{map}(\text{int}, ProdTau, \text{tau}=0..t) \\ &SolNoHom := \left[ -2 + \frac{2e^{2t}}{3} + \frac{13e^{-t}}{12} - \frac{e^{-2t}}{15} + \frac{3e^{3t}}{20} + \frac{e^t}{6}, -2 + \frac{2e^{2t}}{3} + \frac{e^{-2t}}{15} \right. \\ &\quad \left. + \frac{13e^{-t}}{12} - \frac{e^t}{6} + \frac{7e^{3t}}{20}, \frac{4e^{2t}}{3} - \frac{13e^{-t}}{12} - \frac{e^t}{2} + \frac{e^{3t}}{4} \right] \end{aligned} \quad (16)$$

$$\begin{aligned} &> x(t) = SolNoHom[1]; y(t) = SolNoHom[2]; z(t) = SolNoHom[3] \\ &x(t) = -2 + \frac{2e^{2t}}{3} + \frac{13e^{-t}}{12} - \frac{e^{-2t}}{15} + \frac{3e^{3t}}{20} + \frac{e^t}{6} \end{aligned}$$

$$y(t) = -2 + \frac{2e^{2t}}{3} + \frac{e^{-2t}}{15} + \frac{13e^{-t}}{12} - \frac{e^t}{6} + \frac{7e^{3t}}{20}$$

$$z(t) = \frac{4e^{2t}}{3} - \frac{13e^{-t}}{12} - \frac{e^t}{2} + \frac{e^{3t}}{4} \quad (17)$$

$$\begin{aligned} &> \text{ComprobacionCuatro} := x(0) = \text{simplify}(\text{subs}(t=0, \text{SolNoHom}[1])) \\ &\quad \text{ComprobacionCuatro} := x(0) = 0 \end{aligned} \quad (18)$$

$$\begin{aligned} &> \text{ComprobacionCinco} := y(0) = \text{simplify}(\text{subs}(t=0, \text{SolNoHom}[2])) \\ &\quad \text{ComprobacionCinco} := y(0) = 0 \end{aligned} \quad (19)$$

$$\begin{aligned} &> \text{ComprobacionSeis} := z(0) = \text{simplify}(\text{subs}(t=0, \text{SolNoHom}[3])) \\ &\quad \text{ComprobacionSeis} := z(0) = 0 \end{aligned} \quad (20)$$

$$\begin{aligned} &> \text{SolGralFinal} := \text{evalm}(\text{SolHom} + \text{SolNoHom}) \\ \text{SolGralFinal} &:= \left[ -\frac{11e^{-t}}{12} + \frac{14e^{2t}}{3} + \frac{59e^{-2t}}{15} - 2 + \frac{3e^{3t}}{20} + \frac{e^t}{6}, -\frac{11e^{-t}}{12} - \frac{59e^{-2t}}{15} \right. \\ &\quad \left. + \frac{14e^{2t}}{3} - 2 - \frac{e^t}{6} + \frac{7e^{3t}}{20}, \frac{28e^{2t}}{3} + \frac{11e^{-t}}{12} - \frac{e^t}{2} + \frac{e^{3t}}{4} \right] \end{aligned} \quad (21)$$

$$\begin{aligned} &> \text{SolFinUno} := x(t) = \text{SolGralFinal}[1] \\ \text{SolFinUno} &:= x(t) = -\frac{11e^{-t}}{12} + \frac{14e^{2t}}{3} + \frac{59e^{-2t}}{15} - 2 + \frac{3e^{3t}}{20} + \frac{e^t}{6} \end{aligned} \quad (22)$$

$$\begin{aligned} &> \text{SolFinDos} := y(t) = \text{SolGralFinal}[2] \\ \text{SolFinDos} &:= y(t) = -\frac{11e^{-t}}{12} - \frac{59e^{-2t}}{15} + \frac{14e^{2t}}{3} - 2 - \frac{e^t}{6} + \frac{7e^{3t}}{20} \end{aligned} \quad (23)$$

$$\begin{aligned} &> \text{SolFinTres} := z(t) = \text{SolGralFinal}[3] \\ \text{SolFinTres} &:= z(t) = \frac{28e^{2t}}{3} + \frac{11e^{-t}}{12} - \frac{e^t}{2} + \frac{e^{3t}}{4} \end{aligned} \quad (24)$$

$$\begin{aligned} &> \text{Sistema}[1]; \text{Sistema}[2]; \text{Sistema}[3] \\ &\quad \frac{d}{dt} x(t) = -x(t) + y(t) + z(t) + e^t \\ &\quad \frac{d}{dt} y(t) = x(t) - y(t) + z(t) + e^{3t} \\ &\quad \frac{d}{dt} z(t) = x(t) + y(t) + z(t) + 4 \end{aligned} \quad (25)$$

$$\begin{aligned} &> \text{ComprobacionSiete} := \text{simplify}(\text{eval}(\text{subs}(x(t) = \text{rhs}(\text{SolFinUno}), y(t) = \text{rhs}(\text{SolFinDos}), z(t) \\ &\quad = \text{rhs}(\text{SolFinTres}), \text{lhs}(\text{Sistema}[1]) - \text{rhs}(\text{Sistema}[1]) = 0))) \\ &\quad \text{ComprobacionSiete} := 0 = 0 \end{aligned} \quad (26)$$

$$\begin{aligned} &> \text{ComprobacionOcho} := \text{simplify}(\text{eval}(\text{subs}(x(t) = \text{rhs}(\text{SolFinUno}), y(t) = \text{rhs}(\text{SolFinDos}), z(t) \\ &\quad = \text{rhs}(\text{SolFinTres}), \text{lhs}(\text{Sistema}[2]) - \text{rhs}(\text{Sistema}[2]) = 0))) \\ &\quad \text{ComprobacionOcho} := 0 = 0 \end{aligned} \quad (27)$$

$$\begin{aligned} &> \text{ComprobacionNueve} := \text{simplify}(\text{eval}(\text{subs}(x(t) = \text{rhs}(\text{SolFinUno}), y(t) = \text{rhs}(\text{SolFinDos}), z(t) \\ &\quad = \text{rhs}(\text{SolFinTres}), \text{lhs}(\text{Sistema}[3]) - \text{rhs}(\text{Sistema}[3]) = 0))) \\ &\quad \text{ComprobacionNueve} := 0 = 0 \end{aligned} \quad (28)$$

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> restart
> Sistema := diff(x[1](t), t) = x[2](t), diff(x[2](t), t) = -2·x[2](t) - 5·x[3](t) + 3,
    diff(x[3](t), t) = x[2](t) + 2·x[3](t) : Sistema[1]; Sistema[2]; Sistema[3]

$$\frac{d}{dt} x_1(t) = x_2(t)$$


$$\frac{d}{dt} x_2(t) = -2 x_2(t) - 5 x_3(t) + 3$$


$$\frac{d}{dt} x_3(t) = x_2(t) + 2 x_3(t) \quad (29)$$

=
> CondIni := x[1](0) = 0, x[2](0) = 0, x[3](0) = 1
    CondIni := x_1(0) = 0, x_2(0) = 0, x_3(0) = 1 \quad (30)
=
> AA := array([ [0, 1, 0], [0, -2, -5], [0, 1, 2] ])

$$AA := \begin{bmatrix} 0 & 1 & 0 \\ 0 & -2 & -5 \\ 0 & 1 & 2 \end{bmatrix} \quad (31)$$

=
> BB := array([0, 3, 0])

$$BB := \begin{bmatrix} 0 & 3 & 0 \end{bmatrix} \quad (32)$$

=
> Cond := array([0, 0, 1])

$$Cond := \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \quad (33)$$

=
> with(linalg) :
> MatExp := exponential(AA, t)

$$MatExp := \begin{bmatrix} 1 & \sin(t) + 2 \cos(t) - 2 & 5 \cos(t) - 5 \\ 0 & -2 \sin(t) + \cos(t) & -5 \sin(t) \\ 0 & \sin(t) & 2 \sin(t) + \cos(t) \end{bmatrix} \quad (34)$$

=
> SolHom := evalm(MatExp &* Cond)

$$SolHom := \begin{bmatrix} 5 \cos(t) - 5 & -5 \sin(t) & 2 \sin(t) + \cos(t) \end{bmatrix} \quad (35)$$

=
> SolUno := x[1](t) = SolHom[1]

$$SolUno := x_1(t) = 5 \cos(t) - 5 \quad (36)$$

=
> SolDos := x[2](t) = SolHom[2]

$$SolDos := x_2(t) = -5 \sin(t) \quad (37)$$

=
> SolTres := x[3](t) = SolHom[3]

$$SolTres := x_3(t) = 2 \sin(t) + \cos(t) \quad (38)$$

=
> ComprobarUno := x[1](0) = simplify(subs(t=0, SolHom[1]))

$$ComprobarUno := x_1(0) = 0 \quad (39)$$

=
> ComprobarDos := x[2](0) = simplify(subs(t=0, SolHom[2]))

$$ComprobarDos := x_2(0) = 0 \quad (40)$$

=
> ComprobarTres := x[3](0) = simplify(subs(t=0, SolHom[3]))

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$$ComprobarTres := x_3(0) = 1 \quad (41)$$

> CondIni

$$x_1(0) = 0, x_2(0) = 0, x_3(0) = 1 \quad (42)$$

> MatExpTau := map(rcurry(eval, t = t - tau'), MatExp)

$$MatExpTau := \begin{bmatrix} 1 & \sin(t - t) + 2 \cos(t - t) - 2 & 5 \cos(t - t) - 5 \\ 0 & -2 \sin(t - t) + \cos(t - t) & -5 \sin(t - t) \\ 0 & \sin(t - t) & 2 \sin(t - t) + \cos(t - t) \end{bmatrix} \quad (43)$$

> BBtau := array([0, 3, 0])

$$BBtau := \begin{bmatrix} 0 & 3 & 0 \end{bmatrix} \quad (44)$$

> ProdTau := evalm(MatExpTau &\* BBtau)

$$ProdTau := \begin{bmatrix} 3 \sin(t - t) + 6 \cos(t - t) - 6 & -6 \sin(t - t) + 3 \cos(t - t) & 3 \sin(t - t) \end{bmatrix} \quad (45)$$

> SolNoHom := map(int, ProdTau, tau = 0 .. t)

$$SolNoHom := \begin{bmatrix} 3 + 6 \sin(t) - 3 \cos(t) - 6t & -6 + 6 \cos(t) + 3 \sin(t) & 3 - 3 \cos(t) \end{bmatrix} \quad (46)$$

> SolNoUno := x[1](t) = SolNoHom[1]

$$SolNoUno := x_1(t) = 3 + 6 \sin(t) - 3 \cos(t) - 6t \quad (47)$$

> SolNoDos := x[2](t) = SolNoHom[2]

$$SolNoDos := x_2(t) = -6 + 6 \cos(t) + 3 \sin(t) \quad (48)$$

> SolNoTres := x[3](t) = SolNoHom[3]

$$SolNoTres := x_3(t) = 3 - 3 \cos(t) \quad (49)$$

> ComprobacionCuatro := simplify(subs(t = 0, SolNoUno))

$$ComprobacionCuatro := x_1(0) = 0 \quad (50)$$

> ComprobacionCinco := simplify(subs(t = 0, SolNoDos))

$$ComprobacionCinco := x_2(0) = 0 \quad (51)$$

> ComprobacionSeis := simplify(subs(t = 0, SolNoTres))

$$ComprobacionSeis := x_3(0) = 0 \quad (52)$$

> SolGralFinal := evalm(SolHom + SolNoHom)

$$SolGralFinal := \quad (53)$$

$$\begin{bmatrix} 2 \cos(t) - 2 + 6 \sin(t) - 6t & -2 \sin(t) - 6 + 6 \cos(t) & 2 \sin(t) - 2 \cos(t) + 3 \end{bmatrix}$$

> SolFinUno := x[1](t) = SolGralFinal[1]

$$SolFinUno := x_1(t) = 2 \cos(t) - 2 + 6 \sin(t) - 6t \quad (54)$$

> SolFinDos := x[2](t) = SolGralFinal[2]

$$SolFinDos := x_2(t) = -2 \sin(t) - 6 + 6 \cos(t) \quad (55)$$

> SolFinTres := x[3](t) = SolGralFinal[3]

$$SolFinTres := x_3(t) = 2 \sin(t) - 2 \cos(t) + 3 \quad (56)$$

> Sistema[1]; Sistema[2]; Sistema[3]

$$\frac{d}{dt} x_1(t) = x_2(t)$$

$$\frac{d}{dt} x_2(t) = -2 x_2(t) - 5 x_3(t) + 3$$

$$\frac{d}{dt} x_3(t) = x_2(t) + 2 x_3(t) \quad (57)$$

$$\begin{aligned} &> \text{ComprobacionSiete} := \text{simplify}(\text{eval}(\text{subs}(x[1](t) = \text{rhs}(\text{SolFinUno}), x[2](t) \\ &\quad = \text{rhs}(\text{SolFinDos}), x[3](t) = \text{rhs}(\text{SolFinTres}), \text{lhs}(\text{Sistema}[1]) - \text{rhs}(\text{Sistema}[1]) = 0))) \\ &\quad \text{ComprobacionSiete} := 0 = 0 \end{aligned} \quad (58)$$

$$\begin{aligned} &> \text{ComprobacionOcho} := \text{simplify}(\text{eval}(\text{subs}(x[1](t) = \text{rhs}(\text{SolFinUno}), x[2](t) \\ &\quad = \text{rhs}(\text{SolFinDos}), x[3](t) = \text{rhs}(\text{SolFinTres}), \text{lhs}(\text{Sistema}[2]) - \text{rhs}(\text{Sistema}[2]) = 0))) \\ &\quad \text{ComprobacionOcho} := 0 = 0 \end{aligned} \quad (59)$$

$$\begin{aligned} &> \text{ComprobacionNueve} := \text{simplify}(\text{eval}(\text{subs}(x[1](t) = \text{rhs}(\text{SolFinUno}), x[2](t) \\ &\quad = \text{rhs}(\text{SolFinDos}), x[3](t) = \text{rhs}(\text{SolFinTres}), \text{lhs}(\text{Sistema}[3]) - \text{rhs}(\text{Sistema}[3]) = 0))) \\ &\quad \text{ComprobacionNueve} := 0 = 0 \end{aligned} \quad (60)$$

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